DOON GLOBAL SCHOOL

PRE-BOARD-2 EXAMINATIONS (2023-24)

CLASS - XII

SUBJECT- MATHEMATICS(041)

TIME – 3 Hrs M. MARKS- 80

GENERAL INSTRUCTIONS:

1. This question paper contains- five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 20 MCQs carrying 1 mark each.
3. Section B has 5 questions of 2 marks each.
4. Section C has 6 questions of 3 marks each.
5. Section D has 4 questions of 5 marks each.
6. Section E has 3 case based integrated units of assessment (4 marks each)

SECTION-A

(Section A consist of 20 questions of 1 mark each)

Q1 Let A= {3,5}. Then number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

Q2. The range of $\sin^{-1} x$ is

a) $(-\frac{\pi}{2}, \frac{\pi}{2})$ b) $(-\pi, \pi)$ c) $(-\infty, 0)$ d) $(0, \infty)$

Q3. The value of $sin\left[\frac{\pi}{3} + \sin^{-1}\frac{1}{2}\right]$ is equal to

(a)1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Q4 If for a square matrix A, $A^2 - A + I = 0$ then A^{-1} equal

(a) A b) A + I c) $I - A$ d) $\mathrm{A} - \mathrm{I}$

Q5 The co-factor of (-1) in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is

(a)1 (b) 2 (c) -1 (d) 0

Q6. If A =$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, B= $\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and A = B^2, then x equals

(a) ± 1 (b) -1 (c) 1 (d) 2

Q7. The derivative of x^{2x} w.r.t.x is

(a) x^{2x-1} (b) $2x^{2x} logx$ (c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

Q8. The function f(x) = [x], where [x] denotes the greatest integer less than or equal to x, is continuous at

(a) 1 (b) 1.5 (c) -2 (d) 4

Q9. If $x = A\cos 4t + b\sin 4t$,then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) - x (c) 16x (d) -16x

Q10. The interval in which the function f(x) =$2x^3+9x^2+12x-1$ is decreasing, is

(a) (-1, ∞) (b) (-2, -1) (c) (-∞, -2) (d) [-1,1]

Q11. $\int \frac{\sec x}{\sec x - \tan x} dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$ (c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

Q12. $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$ *is equal to*

(a) 1 (b) -1 (c) 2 (d) -2

Q13. The sum of order and the degree of the differential equation $\frac{d}{dx}\left((\frac{dy}{dx})^3\right)$

(a) 2 (b) 3 (c) 5 (d) 0

Q14. Two vectors $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} - a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} - b_3\hat{k}$ are collinear if

(a) $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$ (b) $a_1 = b_1, a_2 = b_2, a_3 = b_3$ (c) $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$ (d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

Q15. The magnitude of the vector $6\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ is

(a) 1 (b) 5 (c) 7 (d) 12

Q16.If for any two events A and B. P(A)= 4/5 and P (A$\cap B) = 7/10$, then P(B/A) is equal to

(a) 1/10 (b) 1/8 (c) 7/8 (d) 17/20

Q17.Five fair coins are tossed simultaneously. The probability of the events that at least one head comes up is

(a) 27/32 (b) 5/32 (c) 31/32 (d) 1/32

Q18. The angle between the lines 2x = 3y = -z and 6x = -y = -4z is

(a) 0^0 (b) 30^0 (c) 90^0 (d) 45^0

DIRECTIONS: In the question number 19 and 20 a statement of assertion (A) is followed by a statement of reason (R). Choose the correct option as:

(a) Both assertion (A) and Reason (R) are true and reason (R) is correct explanation of assertion(A)

(b) Both assertion (A) and Reason (R) are true but reason is not correct explanation of assertion.

(c) Assertion(A) is true but reason (R) is false. (d) Assertion(A) is false but reason (R) is true.

Q19. Assertion: (A): Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least head comes up, 1/3.

Reason (R): Let E and F be two events with a random experiment, then P(F/E)=$\frac{P(E\cap F)}{P(E)}$

Q20.Assertion: The area of parallelogram with diagonals $\vec{a}$ *and* $\vec{b}$ is $\frac{1}{2}[\vec{a} \times \vec{b}]$'

Reason (R): If$\vec{a}$ *and* $\vec{b}$ **represents the adjacent side of a triangle. Then the area of triangle can be obtained by evaluating |$\vec{a} \times \vec{b}$|**

SECTION-B

(Section B consists of 5 questions of 2 marks each)

Q21.Express $\tan^{-1}(\frac{x}{\sqrt{a^2-x^2}})$

Q22. $If\ A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$**, them show that |2A|= 4|A|**

Q23.Find $\int \frac{x+3}{x^2-2x-5}$dx.

Q24. Solve $x\frac{dy}{dx} - y = x^2$

Q25. Find the angle between the vectors $\vec{a} = \hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$

OR

The x- coordinate of a point on the line joining the points P(2,2,1) and Q(5,1, -2) is 4. Find its z-coordinate.

SECTION-C

(Section C consists of 6 questions of 3 marks each)

Q26. .Let R be a relation on the set A of ordered pairs of positive integers defined as $(x, y)R(u, v)$ if and only if$xv = yu$. Show that, R is an equivalence relation

Q27 Differentiate $log(x^{sinx} + cot^2x)$ $with\ respect\ to\ x$

Q28. Find $\int \sqrt{1 - 4x - x^2}$dx.

OR

Evaluate $\int_0^{\pi} \frac{x\ sinx}{1+cos^2x} dx$

Q29. If $y = (logx)^x + x^{logx}$, then find $\frac{dy}{dx}$.

Q30. Find the area of the region bounded by the curves $x^2 + y^2 + 4, y = \sqrt{3x}$ and x-axis in the first quadrant.

Q31. Find the shortest distance between the parallel lines whose vector equation are $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$; $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(4\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

SECTION-D

(Section D consists of 4 questions of 5 marks each)

Q32. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is 8 cu. Cm. If building of tank costs Rs. 70 per sq metre for the base and Rs. 45 per sq metre for sides, what is the cost of least expensive tank?

Q33. If $y = x\ log\frac{x}{a+bx}$, then show that $x^3\frac{d^2y}{dx^2} = (x\frac{dy}{dx} - y)^2$

OR

Find whether the following function f is differentiable at x = 1 and x = 2 or not

$$f(x) = \begin{Bmatrix} x, if\ x < 1 \\ 2 - x, if\ 1 \le x \le 2 \\ -2 + 3x - x^2, if\ x > 2 \end{Bmatrix}$$

Q34. Solve the following linear programming problem graphically. Maximise: $z = 5x + 3y$, subject to constraints $2x + 5y \le 15, 5x + 2y \le 10, x \ge 0, y \ge 0$

Q35. Show that the differential equation $\left[xsin^2 \frac{y}{x} - y\right] dx + x\,dy = 0$ homogeneous. Find the particular solution of this differential equation, given that $y = \frac{\pi}{4}$, $when\ x = 1$

SECTION-E [20 MARKS]

(CASE STUDY BASED QUESTIONS)

(Section E consists of 3 questions. All are compulsory)

Q36. Atrust having a fund of Rs. 30000 invest into two different types of bonds. The first bond pays 5% interest per annum which will be given to' Cancer Aid Society' an NGO. The trust wishes to divide Rs. 30000 among two types of bonds in such a way that they earn an annual total interst of Rs. 1800.

Based on above information, answer the following questions.

(i) **If the amount invested in first bond by Rs. x and in second bond is Rs. y, then what is the system of equation formed?**

(ii) **Write the system of equation in matrix form.**

(iii) **Find the values of x and y.**

OR

What is the inverse of the matrix $\begin{bmatrix} 1 & 1 \\ 5 & 7 \end{bmatrix}$

Q37.In a town, it's rainy one third of days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$. Given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If it's rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it, is not rainy and there is no heavy traffic, In other situations (rainy and no heavy traffic, not rainy and heavy traffic), the probability of being late is $\frac{1}{4}$. You pick a random day.

Based on above information, answer the following questions.

(i) **What is the probability that it's not raining and there is heavy traffic and I am not late?**

(ii) **What is the probability that I am late?**

(iii) **Given that I arrived late at work, what is the probability that it rained that day?**

Q38. A company is into creating a packaging of items. They were into making of a cylindrical shaped container. They tried different options i.e. different sizes of right circular cylinder to get the best option. The cylinder is of the volume 432π ml and should have minimum surface area.Based on above information, answer the following questions.

(i) **What is the radius r of the cylinder for minimum surface area?**

(ii) **What is the realtion between the radius of base and height of cylinder of minimum surface area.**