DOON GLOBAL SCHOOL
HALF YEARLY EXAMINATIONS (2024-25)
CLASS - XI
SUBJECT- MATHEMATICS (041)

TIME – 3Hrs **M.M - 80**

INSTRUCTIONS:

1. **This question paper contains- five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.**
2. **Section A has 20 MCQs carrying 1 mark each.**
3. **Section B has 5 questions of 2 marks each.**
4. **Section C has 6 questions of 3 marks each.**
5. **Section D has 4 questions of 5 marks each.**
6. **Section E has 3 case based integrated units of assessment (4 marks each)**

SECTION-A
(Section A consists of 20 questions of 1 mark each)

Q1. Let S be the set of all real numbers and let R be a relation on S, defined by a Rb ⇔ (1 + ab) > 0. Then, R is

a) Symmetric and Reflexive but not transitive b) Reflexive and transitive but not symmetric

c) Symmetric and transitive but not reflexive d) reflexive and symmetric but not transitive

Q2. Identity relation R on a set A is

a) Reflexive only b) Transitive only c) Equivalence d) Symmetric only

Q3. The principal value of $\sec^{-1}(\frac{2}{\sqrt{3}})$ is

a) $\frac{2\pi}{3}$ b) $\frac{-\pi}{3}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

Q4. Suppose 3 ×3 matrix A=a_{ij} , whose elements are given by, $a_{ij} = i^2 - j^2$ then a_{32} is equal to

a) 5 b) 3 c) 2 d) 1

Q5. If A=$\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then det (adj A) equals

a) a^2 b) a^6 c) a^9 d) a^{27}

Q6. Let y be a function of x such that log (x + y) - 2xy = 0, then y'(0) is

a) 0 b) 1 c) 1/2 d) 3/2

Q7. The function f(x) = x – [x], where **[.]** denotes the greatest integer function is

a) continuous at non-integer points only b) continuous everywhere

c) continuous at integer points only d) differentiable everywhere

Q8. Assume X, Y, Z, W and P are matrices of order 2× n, 3× k, 2 ×p, n ×3 and p× k, respectively. If n = p, then the order of the matrix 7X - 5Z is:

a) 2 ×n b) p ×2 c) p× n d) n ×3

Q9. The differential coefficient of sin (cos (x^2)) w.r.t. x is

a) -2xsin (x^2) cos (cos (x^2)) b) 2xsin(x^2) cos(x^2))

c) 2xcos(x^2) sin(x^2) d) 2xsin(x^2) cos(x^2) cos x

Q10. $\int sin^2x$ **dx is equal to:**

a) $\frac{x}{2}+\frac{sin2x}{4}+c$ b) $\frac{x}{2}-\frac{sin2x}{4}+c$ c) $\frac{x}{2}+\frac{sin2x}{2}+c$ *d)* $\frac{x}{2}-\frac{sin2x}{2}+c$

Q11. A linear programming problem deals with the optimization of a/an :

a) logarithmic function b) linear function c) quadratic function d) exponential function

Q12. The number of corner points of the feasible region determined by constraints x ≥ 0, y ≥ 0, x + y ≥ 4 is:

a) 0 b) 1 c) 2 d) 3

Q13. Derivative of $e^{2x}$with respect to e^x,is

a)e^x b) $2e^x$ c)$2e^{2x}$ d)$2e3^x$

Q14. Given that $[1 \quad x]\begin{bmatrix}4 & 0\\ -2 & 0\end{bmatrix}$

a) -4 b) -2 c) 2 d) 4

Q15. The value of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{18}$

Q16. A and B are skew-symmetric matrices of same order. AB is symmetric, if :

(a) AB = O (b) AB = BA (c) AB = BA (d) BA = O

Q17. $\int 2^{x+2}\,dx$ is equal to:

a) $2^{x+2}+C$ *b)* $2^{x+2}log2+C$ *c)* $)\frac{2^{x+2}}{log2}+C$ *d)2* $\frac{2^x}{log2}+C$

Q18. If A = $\begin{bmatrix}1 & 4 & x\\ z & 2 & y\\ -3 & -1 & 3\end{bmatrix}$is a symmetric matrix, then the value of x + y + z

(a)10 (b) 6 (c) 8 (d) 0

DIRECTIONS: In the question number 19 and 20 a statement of assertion (A) is followed by a statement of reason (R). Choose the correct option as:

(a) Both assertion (A) and Reason (R) are true and reason (R) is correct explanation of assertion(A)

(b) Both assertion (A) and Reason (R) are true but reason is not correct explanation of assertion.

(c) Assertion(A) is true but reason (R) is false.

(d) Assertion(A) is false but reason (R) is true.

Q19.Assertion The Relation R in the set A= {1,2,3,4,5,6} as R= {(x, y): y is divisible by x} is an equivalence Relation

Reason: A relation R on the set A is equivalence if it is reflexive, symmetric and transitive

Q20.Assertion: Domain of $\cos^{-1}x$ is [-1,1]

Reason: The range of principal value branch is y = $\cos^{-1}x$ is [0, π] – $\{\frac{\pi}{2}\}$

SECTION- B

(Section B consist of 5 questions of 2 marks each)

Q21. Find the value of $\boldsymbol{tan^{-1}(1)+cos^{-1}(-\frac{1}{2})+sin^{-1}(-\frac{1}{2})}$

Q22. Show that the function given by f(x) = sin x is

i strictly increasing in (0, $\frac{\pi}{2}$)

ii. strictly decreasing in ($\frac{\pi}{2}$,π)

Q23. Find: $\int (cos^2 2x – sin^2 2x)dx$

Q24. Show that the function f(x) = $4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

Q25. The volume of a cube is increasing at the rate of $6cm^3/s$. How fast is the surface area of cube increasing, when the length of and edge is 8cm?

SECTION-C

(Section C consists of 6 questions of 3 marks each)

Q26. Find: $\int \frac{sec^2 x}{\sqrt{tan^2 x+4}}\, dx$

Q27. A relation R on set A = {1, 2, 3, 4, 5} is defined as R = {(x, y) : $|x^2 - y^2| < 8$}. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from R ⟶ R as f(x) = ax + b, such that f (1) = 1and f (2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

Q28. Solve the following linear programming problem graphically:

Maximize z = 500x + 300y,

subject to constraints

x + 2y≤ 12

2x + y ≤ 12

4x + 5y ≥ 20

x ≥ 0, y ≥ 0

Q29. Find $\int \frac{3x+5}{\sqrt{x^2+2x+4}}\, dx$

Q30.Find the value of x $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0$

Q31. Prove that the function f is given by f(x) = | x- 1|, x∈ R is not differentiable at x =1.

SECTION- D

(Section D consist of 4 questions of 5 marks each)

Q32. A square piece of tin of side 18cm is to be made into a box without top, by cutting -off square from each corner and folding up the flaps of the box. What should be the side of the square to be cut off so that the volume of the box is maximum possible.

Q33. If A= $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Using A^{-1} solve ths system of equation 2x - 3y + 5z =11, 3x + 2y - 4z = -5, x+y-2z = -3

Q34. Find $\int \sqrt{1 - 4x - x^2}$dx.

Q35. **If $y = (logx)^x + x^{logx}$, then find $\frac{dy}{dx}$.**

SECTION-E

(Case study-based questions are compulsory).

Q36. A relation R on a set A is said to be an equivalence relation on A iff it is

Reflexive i.e., (a, a) ∈ R ∀ a ∈ A.

Symmetric i.e., (a, b) ∈ R ⇒(b, a) ∈ R ∀ a, b∈ A.

Transitive i.e., (a, b) ∈ R and (b, c) ∈ R {a, c) ∈ R ∀ a, b, c ∈ A.

a) If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A - {1, 2, 3}, then define if R is reflexive, symmetric or transitive.

(b) If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then check whether R is reflexive, symmetric or transitive

(c) If the relation R on the set N of all natural numbers defined as R - {(x, y) : y = x + 5 and x < 4}, then check whether R is reflexive, symmetric or transitive.

(d) If the relation R on the set A = {1, 2, 3, ... 13, 14} defined as R = {(x, y) : 3x - y = 0}, then define R.

Q37. Ankit wants to construct a rectangular tank for his house that can hold 80 ft^3 of water. He wants to construct on one corner of terrace so that sufficient space is left after construction of tank. For that he has to keep width of tank constant 5ft, but the length and heights are variables. The top of the tank is open. Building the tank cost ₹20 per sq. foot for the base and ₹10 per sq. foot for the side.

(a) Express cost of tank as a function of height(h).

(b) Verify by second derivative test that cost is minimum at critical point.

(c) Find the value of h at which c(h) is minimum.

(d) Find the minimum cost of tank?

Q38. A Trust fund has rupees 30000 that must be invested in two different types of bones, the first bone pays 5% interest per year and the second bond pay 7% interest per year. Using matrix multiplication, determine how to divide rupees 30000 among the two types of bonds in the trust fund must obtain an annual total interest of (a) rupees 1800 (b) rupees 2000.